tension within the elastic layer, N/m^2; M_i, vibrating mass of the elastomer, $kg \cdot sec^2/m^3$; ρ, density of the elastomer, kg/m^3; E_i, modulus of elasticity, N/m^2; $E_i' = E_i/h_i$, rigidity of the elastic layer, N/m^3; i, number of plate layer; $E = E' + E''$, complex modulus of elasticity; $E'(\omega)$, modulus of elasticity; $E''(\omega)$, loss modulus; $\tan \varphi = E''/E'$, energy-loss coefficient (the loss tangent); P, surface pulsation pressure, N/m^2; σ_{bend}, bending stress in the plate, N/m^2; G, shear modulus, N/m^2; η_i, viscosity of the elastic layer, $N \cdot sec/m^2$; λ, Lamé parameter; $K = \lambda + {}^2/_3 G$, volumetric modulus of elasticity, N/m^2; $\eta_k = G\tau_k$, shear viscosity, $N \cdot sec/m^2$; $\tilde{E}$, portion of the boundary-layer energy accumulated in the elastomer; Φ, dissipated portion of the energy; $r_1 = \tau\Phi/\tilde{E}$, coefficient of absorption for potential energy; $r_2 = (\tilde{E} - \tau\Phi)/\tilde{E}$, coefficient of the potential energy of the boundary layer accumulated in the elastomer; C_x, coefficient of frictional resistance; $\xi = (C_{x_{rigid}} - C_{x_{elast}})C_{x_{rigid}}^{-1}$, coefficient of reduction in frictional resistance.

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INVESTIGATION OF EHD FLOW BASED ON A NUMERICAL SOLUTION OF THE NAVIER–STOKES EQUATIONS

L. P. Pasechnik and I. V. Ufatov

UDC 532.516

The proposed numerical method is used to examine the physical pattern of EHD flow at a high-voltage flat electrode.

Electrohydrodynamic (EHD) flows of low-conductivity liquids from a high-voltage electrode have been studied both theoretically and experimentally in numerous works (see, for example, [1-4]). Despite a superficial similarity in the EHD flow patterns and those of a Landau–Slezkin "submerged jet," the latter cannot be treated as a sufficiently exact model of EHD flow. In particular, in the case of an immersed electrode it makes no provision for the effect of the friction of the jet against the wall of the vessel. An estimate is presented in [1] of the possible velocity of isothermal electrical convection as part of a study of the laminar flow of an incompressible dielectric liquid around an electrode. According to [1, 2] the flow is caused primarily by a Coulomb force acting on the space charge that is formed because of a nonuniformity in electrical conductivity that is weak but different from zero. A semiempirical formula was proposed in [1] for a steady-state charge, and by means of this formula it became possible qualitatively to describe the phenomena of electrical con-

Patrice Lumumba Peoples' Friendship University, Moscow.

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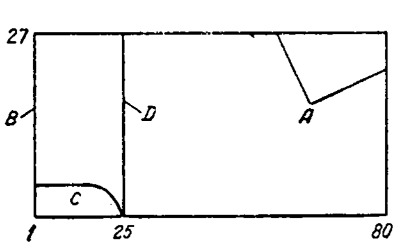

Fig. 1

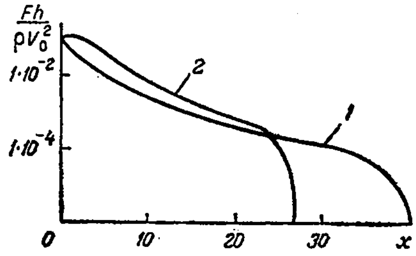

Fig. 2

Fig. 1. Theoretical model of the flow region: A) counterelectrode; B) conditional boundary of field uniformity; C) flat electrode; D) Plexiglas vessel lid.

Fig. 2. Distribution of repulsive component of the EHD force at a voltage of 3.5 kV: 1) at the axis; 2) transverse distribution at a distance of 0.1 cm from the electrode. x, mm.

vection, but the calculated values of velocity are several times greater than the experimental values.

In this paper, on the basis of the proposed numerical method of solving the Navier–Stokes equations with the aid of the Green's function, we have obtained the physical pattern of EHD flow in transformer oil at a high-voltage flat electrode embedded into a nonconducting wall. The calculated values of velocities are in good agreement with experimental data.

The Navier–Stokes equations are solved by means of the Green's function in which consideration is given to the effect of the external field $\mathbf{F} = -\beta\varepsilon(\mathbf{E}\cdot\nabla)\mathbf{E}$:

$$\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = \nabla p + \mu\nabla^2\mathbf{v} + \mathbf{F}, \quad (1)$$

$$\nabla\cdot\mathbf{v} = 0, \quad (2)$$

where ρ is density; μ is dynamic viscosity; β = 0.3 mm/kV [1]. Applying the div operator to (1), with consideration of (2), we obtain

$$\nabla^2 p = -\rho\,\mathrm{div}(\mathbf{v}\cdot\nabla)\mathbf{v} + \mathrm{div}\,\mathbf{F}. \quad (3)$$

We will take the Green's function for an unbounded region as the Green's function, and we will neglect the integral of the pressure over the boundary because of its smallness in comparison with the integral over the entire region (for additional details see [5]). The obtained equations are solved by a difference method, and the solution is given in the Appendix.

The distribution of the electric potential is sought in the region shown in Fig. 1. We find the potential by solving the Laplace equation $\Delta\Psi = 0$ for given values of Ψ at the boundary of the region, i.e., at the electrodes. The potential equal to zero (the counterelectrode) is given for the surface A; the linear distribution of the potential is given for the surface B. Rounding off of the knife edge gives the nonuniformity of the electric field. The Laplace equation is solved by a grid method (see, for example, [6, 7]). The calculations were carried out for a flat electrode with a curvature radius of 0.5 cm in transformer oil with a viscosity ν = 28 St. Figure 2 shows the distribution of the repulsive component of the EHD force at a voltage of 3.5 kV at the axis and a lateral distribution of the repulsive component of this force at a distance of 0.5 cm from the electrode.

As a result of computer calculations, we obtained the physical pattern of the EHD flow. We can see from Figs. 3 and 4 that the EHD flow is primarily concentrated near the electrode. It is not difficult to see that the velocity, equal to zero at the beginning and end of the curves, corresponds to the absence of EHD flow in the transformer oil at the walls of the vessel (i.e., the condition of "adhesion" is satisfied). The curves exhibit a maximum at a distance from the electrode that is equal approximately to the radius of the electrode. Thus, the maximum value of velocities for voltages of 1.7, 2.1, 3.0, and 3.5 kV, respectively, amount to 1.3, 2.7, 7.8, and 13.4 mm/sec, and in this case the increase in the distance from the electrode, at which the maximum values are found for the calculated flows, is proportional to the value of the voltages, i.e., these distances become greater than the elec-

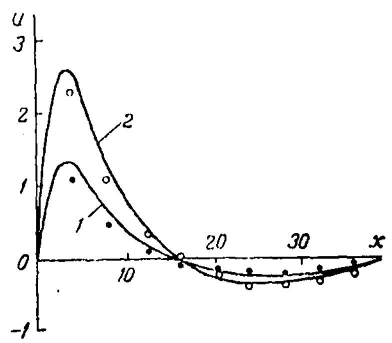

Fig. 3

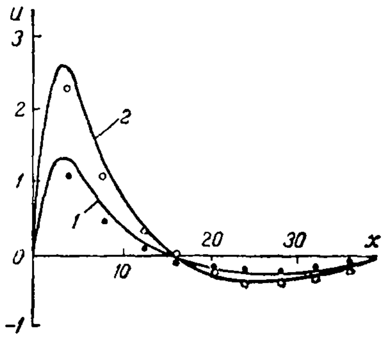

Fig. 4

Fig. 3. Distribution of EHD velocities: 1) at a voltage of 1.7 kV; 2) at a voltage of 2.1 kV; points denote experimental data [3]. u, mm/sec.

Fig. 4. Distribution of EHD velocities: 1) at a voltage of 3.0 kV; 2) at a voltage of 3.5 kV; points denote experimental data [3].

trode radius as the voltage increases. The obtained pattern of EHD flow corresponds to the experimental data [1].

APPENDIX

The original system of equations (in dimensionless variables) has the form:

$$uu_x + vu_y = \frac{1}{\pi}\iint \frac{(u_{x'}^2 + u_{y'}v_{x'} - (F_{1x'} + F_{2y'})/2)(x - x')}{(x - x')^2 + (y - y')^2}\, dx'dy' + \nu\nabla^2 u + F_1, \tag{A.1}$$

$$uv_x + vv_y = \frac{1}{\pi}\iint \frac{(u_{x'}^2 + u_{y'}v_{x'} - (F_{1x'} + F_{2y'})/2)(y' - y')}{(x - x')^2 + (y - y')^2}\, dx'dy' + \nu\nabla^2 v + F_2. \tag{A.2}$$

Here $\nu = \mu/\rho u^0 h$; u^0 and h are the units of velocity and length measurement.

We will obtain the difference equations for some point with coordinates (x_i, y_i) by expansion in the neighborhood of this point of the functions u and v with accuracy to the third-order terms inclusively [6]:

$$u = a + bx + cy + dx^2 + exy + fy^2 + lx^3 + mx^2y + nxy^2 + ty^3. \tag{A.3}$$

The corresponding expression for v is similar in form and the coefficients a, b, c... are replaced by a_1, b_1, c_1... . From the continuity equation we obtain

$$c_1 = -b,\ e = -2f_1,\ 3l = -m_1,\ m = -n_1,\ n = -3t_1,\ e_1 = -2d. \tag{A.4}$$

To these six equations we will add the relationships which associate the values of the functions at the points having coordinates (x_i, y_i), $(x_{x\pm 1}, y_i)$, $(x_i, y_{i\pm 1})$:

$$u_1 = u(x_{i+1}, y_i) = a + b + d + c,\ u_2 = u(x_i, y_{i+1}) = a + c + f + t,$$
$$u_3 = u(x_{i-1}, y_i) = a - b + d - l,\ u_4 = u(x_i, y_{i-1}) = a - c + f - t. \tag{A.5}$$

Having substituted expansion (A.3) into (A.1) and (A.2) and equating the coefficients for identical degrees x_i and y_i, we obtain the system of equations

$$ab + a_1c = 2\nu(d + f) = I + F_1, \tag{A.6}$$

$$ab_1 - a_1b = 2\nu(d_1 + f_1) + J + F_2, \tag{A.7}$$

$$b^2 + b_1c + 2(ad - a_1f_1) + I_x + 6\nu(l - t_1) + F_{1x}, \tag{A.8}$$

$$b^2 + b_1 c + 2(a_1 f_1 - ad) = J_y + 6\nu (t_1 - l) + F_{2y}, \tag{A.9}$$

$$2(ad_1 - a_1 d) = J_x + 2\nu (3l_1 - m) + F_{2x}, \tag{A.10}$$

$$2(a_1 f - f_1 a) = J_y + 2\nu (m + 3t) + F_{1y}. \tag{A.11}$$

Here I, I_x, J... are the values of the integrals and their derivatives in (A.1) and (A.2), taken at the point (x_i, y_i). We will carry out the integration by dividing the region into squares with a side equal to unity, with the exception of the neighborhood of the point (x_i, y_i) where the side of the square is equal to two, and with the center at that point. Thus, $I = \Sigma \alpha_k I_k + \alpha_0 I_0$, $\alpha_k = (u_7 + u_8 - u_5 - u_6)^2 + (u_6 + u_7 - u_8 - u_5)(v_7 + v_8 - v_5 - v_6)$, $u_5 = u(x_k, y_k)$, $u_6 = (u_k, y_{k+1})$, $u_7 = u(x_{k+1}, y_{k+1})$, $u_8 = u(x_{k+1}, y_k)$, where x_k and y_k are the coordinates of the left lower node of the square over which the integration is carried out; $\alpha_0 I_0$ is the contribution at the singular point;

$$I_k = \frac{1}{4\pi} \int_{x_k}^{x_{k+1}} dx' \int_{y_k}^{y_{k+1}} \frac{(x - x')}{(x_i - x')^2 + (y_i - y')^2} dy'.$$

Integrals J_k are obtained from I_k by replacing x with y and by substituting x for y. For the singular point $I_{x0} = J_{y0} = 1/4$, $I_0 = J_0 = 0$.

The system of equations (A.2-A.11), written for each inside point of the grid, is solved by the Seidel method, proceeding from some initial distribution.

Equations (A.6)-(A.7) are used to obtain improved values of a and a_1:

$$a(b + 4\nu) + a_1 c = \nu(u_1 + u_2 + u_3 + u_4)/2 + I + F_1,$$
$$ab_1 + a_1(4\nu - b) = \nu(v_1 + v_2 + v_3 + v_4)/2 + J + F_2.$$

The coefficients b, b_1, c, etc., must be expressed in terms of the values of u and v at the nodes of the grid:

$$b = \frac{1}{4}(u_1 + u_3 - v_2 + v_4) + \frac{I_x'}{12\nu} + \frac{1}{6\nu}(a_1 f_1 - ad) + \frac{F_{1x} - F_{2y}}{24\nu},$$

$$l_1 = \frac{\varkappa + k_1}{2} - \sqrt{\frac{(\varkappa + k_1)^2}{4} + b^2 - \frac{\alpha_0}{16}},$$

$$t = \frac{1}{3\nu}(ad_1 - a_1 d + a_1 f - f_1 a - I_y) - \frac{F_{2x} - F_{1y}}{6\nu} - l_1,$$

$$l = \frac{u_1 - u_3}{2} - b, \quad c_1 = -b, \quad t_1 = \frac{v_2 - v_4}{2} - l_1,$$

$$c = (u_2 - u_4)/2 - t,$$

$$b_1 = \frac{(v_1 - v_3)}{2} - l_1,$$

where I_x' is the integral without any singular-point contribution; α_0 is the value of α_k at the singular point;

$$\varkappa = k - \frac{u_2 - u_4}{2} - \frac{F_{2x} + F_{1y}}{6\nu}; \quad k_1 = \frac{v_1 - v_3}{2};$$

$$k = \frac{1}{3\nu}(ad_1 - a_1 d + a_1 f - f_1 a - I_y); \quad \frac{\alpha_0}{16} = \frac{I_x + J_y + F_{1x} + F_{2y}}{2}.$$

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THE DYNAMICS OF QUASISTEADY FLOW OF A LIQUID–GAS MIXTURE IN A CONDUIT

V. A. Bruk

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The one-dimensional flow of a liquid–gas mixture is investigated theoretically for the case of a horizontal conduit with phase transition. Approximate solutions have been obtained.

The calculation of the nonsteady flows of gases and liquids in tubes is a complex mathematical problem and is usually accomplished by means of numerical methods [1-4]. Solutions in analytical form have been obtained in [5-7] and the basic quantitative relationships governing one-dimensional nonisothermal quasisteady gas flows have been investigated. Below we will examine the one-dimensional quasisteady nonisothermal flow of a liquid–gas mixture in a horizontal tube of constant cross section. Here we will take into consideration the influence of the phase transition on the process being investigated.

The liquid concentration is characterized by the true φ and flowrate β volumetric concentrations [8]. We will assume that the value of φ at the inlet φ_i to the tube is small and, since the process is nearly steady, the quantity φ is also small:

$$\varphi \ll 1. \tag{1}$$

The quantities φ and β are associated by the equality [8]

$$\varphi = \beta v_m / v_\ell . \tag{2}$$

We will examine only the stratification of the flow which is observed at low liquid concentrations $\beta \leq 10^{-2}$ [8]. Since the viscosity of the liquid is considerably greater than the viscosity of the gas and, moreover, the liquid moves near the wall of the tube as the flow becomes separated (in the lower portion), the velocity of the flow v_ℓ must be small in comparison with the gas velocity v. Then

$$\beta \ll \varphi. \tag{3}$$

This assumption is confirmed by experimental results [8]. When $\beta \sim 10^{-2}$ the value of φ is on the order of 10^{-1}. According to these experimental data, however, for the small values of β that we are studying the function $\varphi(\beta)$ is extremely close to the linear. Thus we can assume that

$$\frac{v_\ell}{v_m} = \frac{\beta}{\varphi} = \text{const}. \tag{4}$$

Keeping in mind the smallness of β and φ, instead of (4) we can write

$$\alpha \equiv \frac{v_\ell}{v} = \frac{\beta}{\varphi} = \text{const}. \tag{5}$$

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